Proofs for Midterm

Analysis IV, Spring semester

EPFL, Mathematics section, Prof. Dr. Maria Colombo

This is an exhaustive list of the theorems that you can be asked to state and prove during the final exam.

- For each proof, a reference in the Course Lecture Notes, in Tao's book or Dacorogna's Lecture Notes is mentioned. They are just here in case you don't find in your notes the proof presented in class, you don't have to necessarily read them. In principle, the latters and the references are fairly similar, but there might be small differences in the way they are presented. Both versions are acceptable during the exam.
- The statement of any definition mentioned during the lectures can be asked.
- All other material treated in class can be useful for solving the exercises, but no proof on them will be asked.
- Partial proofs may be asked. For example, in the midterm you were asked to prove only part of Tonelli-Fubini's theorem. Another example could be to state the theorem on the approximation of L^p functions by C_C^∞ functions and prove only the approximation of C^0 functions by C_C^∞ functions.

Measure Theory

- Countable sub-additivity of the outer measure (Lemma 1.2, (v) of the course Lecture Notes, Lemma 7.2.5, (x) in the book of Tao).
- For any closed box B, $m^*(B) = \mathrm{vol}(B)$ (Proposition 1.3 of the course Lecture Notes, Proposition 7.2.6 in the book of Tao).
- Countable additivity of the Lebesgue measure (Lemma 1.11 of the course Lecture Notes, Lemma 7.4.8 in the book of Tao).
- Measurability of the sup, inf or pointwise limit of a sequence of measurable functions (Lemma 1.22 of the course Lecture Notes, Lemma 7.5.10 in the book of Tao).
- Linearity of the Lebesgue integral for simple functions (Proposition 2.5 (ii) and (iii) of the course Lecture Notes, Proposition 8.1.10, (b) and (c) in the book of Tao).
- Lebesgue monotone convergence theorem (Theorem 2.7 of the course Lecture Notes, Theorem 8.2.9 in the book of Tao).
- Fatou's lemma (Lemma 2.10 of the course Lecture Notes, Lemma 8.2.13 in the book of Tao).
- Lebesgue dominated convergence theorem (Theorem 2.14 of the course Lecture Notes, Theorem 8.3.4 in the book of Tao).

- Fubini's theorem (Theorem 2.20 of the course Lecture Notes, Theorem 8.5.1 in the book of Tao).
- Hölder's inequality (Theorem 3.1 of the course Lecture Notes, Theorem 16.27 in Dacorogna's lecture notes).
- Minkowski's inequality (Proposition 3.2 (iv) of the course Lecture Notes, Theorem 16.28 in Dacorogna's lecture notes).
- Every convergent sequence in L^p has a pointwise convergent subsequence (Theorem 3.3 of the course Lecture Notes, Theorem 16.33 in Dacorogna's lecture notes).
- Density of C_C^∞ in L^p (Theorem 3.4 part I,II and III of the course Lecture Notes, Theorem 16.29 in Dacorogna's lecture notes).

Fourier Series, Fourier Transform and PDEs

- Weierstrass Theorem (Theorem 4.6 of the course Lecture Notes, Theorem 5.4.1 in the book of Tao).
- Inversion formula and Parseval's identity for trigonometric polynomials (Corollary 4.4 and 4.5 of the course Lecture Notes, Page 115 and Corollary 5.3.6 in the book of Tao).
- Convergence in L^2 of Fourier series (Theorem 4.9 of the course Lecture Notes, Theorem 17.17 in Dacorogna's lecture notes).
- Parseval's identity for Fourier series (Theorem 4.10 of the course Lecture Notes, Theorem 17.17 in Dacorogna's lecture notes).
- Pointwise convergence of Fourier series (Theorem 4.11 of the course Lecture Notes, Theorem 17.16 in Dacorogna's lecture notes).
- Uniform convergence of Fourier series (Theorem 4.13 of the course Lecture Notes, Theorem 17.18 in Dacorogna's lecture notes).
- Fourier transform of derivative (Lemma 5.1 (iii) of the course Lecture Notes, Theorem 18.2, (iii) in Dacorogna's lecture notes).
- Fourier inversion formula (Theorem 5.4 of the course Lecture Notes, Theorem 18.5 in Dacorogna's lecture notes).
- Plancherel's identity for Fourier transform (Theorem 5.7 of the course Lecture Notes, Theorem 18.7 in Dacorogna's lecture notes).
- Result on the heat kernel and the solution of the heat equation on $\mathbb{R}$ (Section 6.1 and Theorem 6.1 of the course Lecture Notes, section 19.2.2 and 19.2.3 in particular Theorem 19.9 in Dacorogna's lecture notes).